Supplementary Document

This document contains the omitted proofs for theorems and technical lemmas of the paper (not intending for publication). Section SA replicates the proofs of results from Yu et al. (2010), Section SB contains the proofs of all technical lemmas in the appendix, and Section SC contains three extensions on advertising signal, heterogeneous marginal costs, and additive utility function. We replicated the complete statements of the corresponding results so that this supplementary document can be independently read and used as a reference.

Section SA Proofs of Results from Yu et al. (2010)

Lemma 1

$$p^*_{2t}(S) = \max\left(p^U_{2t}, p^B_{2t}(S)\right) = \begin{cases} p^U_{2t} & \text{if } \frac{T-S}{N_1+N_2-S} \geq \overline{G}(p^U_{2t}-t) \\ p^B_{2t}(S) & \text{otherwise} \end{cases} \tag{2}$$

where p^U_{2t} maximizes the unconstrained profit $p_2\overline{G}(p_2-t)$ and $p^B_{2t}(S)$ is the market-clearing price. Specifically,

$$p^U_{2t} \begin{cases} \in (t+\underline{\alpha}, t+\overline{\alpha}) \text{ and is a solution to } p^U_{2t} = \frac{\overline{G}(p^U_{2t}-t)}{g(p^U_{2t}-t)} & \text{if } t < \bar{t} = \frac{1}{g(\underline{\alpha})} - \underline{\alpha} \\ = t+\underline{\alpha} & \text{if } t \geq \bar{t} \end{cases} \tag{3}$$

and $\overline{G}(p^B_{2t}(S)-t) = \min(1, \frac{T-S}{N_1+N_2-S})$.

Proof of Lemma 1 Recall that $p^*_{2t}(S)$ is the maximizer of $\pi_{2t}(p_2,S)$ defined below:

$$\pi_{2t}(p_2,S) = p_2 \min\{T-S, (N_1+N_2-S)\overline{G}(p_2-t)\} \tag{1}$$

The proof is in two steps: (i) the unconstrained profit $p_2\overline{G}(p_2-t)$ is strictly quasi-concave and has a unique maximizer p^U_{2t} as in equation (3); and (ii) $p^*_{2t}(S)$ equals to the maximum of p^U_{2t} and $p^B_{2t}(S)$, as in equation (2).

(i) Since $\overline{G}(\underline{\alpha}) = 1$ and $\overline{G}(\overline{\alpha}) = 0$, it suffices to consider $p_2 \in [t+\underline{\alpha}, t+\overline{\alpha}]$. We divide the proof into two cases depending on the quality level, t:

- ($t < \bar{t} = \frac{1}{g(\underline{\alpha})} - \underline{\alpha}$) Taking derivative with respect to p_2, we have

$$\frac{d[p_2\overline{G}(p_2-t)]}{dp_2} = \overline{G}(p_2-t) - p_2 g(p_2-t) = g(p_2-t)\left[\frac{\overline{G}(p_2-t)}{g(p_2-t)} - p_2\right]$$

Evaluating at two boundary points $p_2 = t+\underline{\alpha}$ and $p_2 = t+\overline{\alpha}^-$, we have $\frac{d[p_2\overline{G}(p_2-t)]}{dp_2}|_{p_2=t+\underline{\alpha}} = 1-(t+\underline{\alpha})g(\underline{\alpha}) > 0$ and $\frac{d[p_2\overline{G}(p_2-t)]}{dp_2}|_{p_2=(t+\overline{\alpha})^-} = 0-(t+\overline{\alpha}^-)g(\overline{\alpha}^-) < 0$. Note that from logconcavity, $\frac{\overline{G}(x)}{g(x)} - x$ is strictly decreasing. Hence, $p_2\overline{G}(p_2-t)$ is strictly quasi-concave in p_2. Thus, the first-order condition, $p^U_{2t} = \frac{\overline{G}(p^U_{2t}-t)}{g(p^U_{2t}-t)}$, results in a unique optimal solution in $(t+\underline{\alpha}, t+\overline{\alpha})$.

- ($t \geq \bar{t}$) It is easy to show that $\frac{d[p_2\overline{G}(p_2-t)]}{dp_2}|_{p_2=t+\underline{\alpha}} \leq 0$. By the strict quasi-concavity above, $\frac{d[p_2\overline{G}(p_2-t)]}{dp_2} \leq 0$ for all $p_2 \in [t+\underline{\alpha}, t+\overline{\alpha}]$. Thus, p^U_2 must be $t+\underline{\alpha}$.

(ii) From the definition of $p^B_2(S)$, we then have

$$\pi_{2t}(p_2,S) = \begin{cases} (T-S)p_2 & \text{for } p_2 \leq p^B_{2t}(S) \\ (N_1+N_2-S)\overline{G}(p_2-t)p_2 & \text{for } p_2 \geq p^B_{2t}(S) \end{cases}$$

Notice that any spot price $p_2 < p^B_{2t}(S)$ cannot be optimal and the objective becomes to choose p_2 maximizing $(N_1+N_2-S)\overline{G}(p_2-t)p_2$, subject to $p_2 \geq p^B_{2t}(S)$. Note that, for a given S, the function, $(N_1+N_2-S)\overline{G}(p_2-t)p_2$ is unimodal and peaks at p^U_{2t}. Hence, if $p^U_{2t} \geq p^B_{2t}(S)$ (equivalent to $(N_1+N_2-S)\overline{G}(p^U_{2t}-t) \leq T-S$), then unconstrained maximum is feasible and $p^*_{2t}(S) = p^U_{2t}$. Otherwise, if $p^U_{2t} < p^B_{2t}(S)$, $(N_1+N_2-S)\overline{G}(p_2-t)p_2$ decreases in p_2 for all $p_2 \geq p^B_{2t}(S)$. Hence, $p^*_{2t}(S) = p^B_{2t}(S)$. □

Corollary 1 *For $S \in [0, \min(T, N_1)]$, $T - S \geq (N_1 + N_2 - S)\overline{G}(p^*_{2t}(S) - t)$.*

Proof of Corollary 1 By Lemma 1, $p^*_{2t}(S) \geq p^B_{2t}(S)$, implying $\overline{G}(p^*_{2t}(S) - t) \leq \overline{G}(p^B_{2t}(S) - t) = \frac{T-S}{N_1+N_2-S}$. □

Theorem 1 *(a) There exist two critical numbers, T_1 and T_D, $0 \leq T_1 \leq T_D \leq N_1 + N_2$, such that*

- *if $T \leq T_1$, then $S^f_t = 0$* **[no advance selling]**,
- *if $T \in (T_1, T_D)$, then $0 < S^f_t < \min(T, N_1)$* **[limited advance selling]**,
- *If $T_D \leq T < N_1 + N_2$ or $T \geq N_1 + N_2$ and $t < \bar{t}$, then $S^f_t = \min(T, N_1)$* **[full advance selling]**,
- *If $T \geq N_1 + N_2$ and $t \geq \bar{t}$, then S^f_t is any value between zero and $\min(T, N_1)$* **[advance selling and spot-only selling are equivalent]**.

Proof of Theorem 1 Recall that for $S \in [0, \min(T, N_1)]$,

$$\pi^f_t(S) = p^*_{1t}(S)S + p^*_{2t}(S)\min\{T - S, (N_1 + N_2 - S)\overline{G}(p^*_{2t}(S) - t)\} \tag{7}$$

Let $S^=_t$ be the *lowest* amount of capacity used in advance at which the market is cleared in the spot period. Clearly, $S^=_t$ is the solution to $\frac{T-S}{N_1+N_2-S} = \overline{G}(p^U_{2t} - t)$ and $S^=_t = \frac{T-(N_1+N_2)\overline{G}(p^U_{2t}-t)}{G(p^U_{2t}-t)}$. Thus, $p^*_{2t}(S) = p^U_{2t}$ for $0 \leq S \leq S^=_t$, and $p^*_{2t}(S) = p^B_{2t}(S)$ for $S \geq S^=_t$.

We define two functions, $f^U_t(S)$ and $f^B_t(S)$, corresponding to the seller's total profits when the market is cleared in spot and when it is not, respectively. For $S \in [0, \min(T, N_1)]$, we have

$$\begin{aligned} f^U_t(S) &:= \mathrm{E}[\min(p^U_{2t}, t + \alpha)]S + (N_1 + N_2 - S)\overline{G}(p^U_{2t})p^U_{2t} \\ &= \mathrm{E}[(t+\alpha)1_{t+\alpha \leq p^U_{2t}}]S + (N_1 + N_2)\overline{G}(p^U_{2t})p^U_{2t} \\ f^B_t(S) &:= \mathrm{E}[\min(p^B_{2t}(S), t + \alpha)]S + (T - S)p^B_{2t}(S), \text{and} \end{aligned}$$

$$\pi^f_t(S) = \begin{cases} f^U_t(S) \text{ for } S \leq S^=_t \text{ and } S \in [0, \min(T, N_1)] \\ f^B_t(S) \text{ for } S \geq S^=_t \text{ and } S \in [0, \min(T, N_1)] \end{cases} \tag{S.1}$$

It is shown in Yu et al. (2010) that $f^B_t(S)$ is strictly quasi-concave in S and has a unique maximizer S^B_t on $[0, \min(T, N_1)]$. Specifically, there exist two thresholds T_1 and T_D, $0 \leq T_1 \leq T^D \leq N_1 + N_2$, such that $S^B_t = 0$ for $0 < T \leq T_1$, $S^B_t \in (0, \min(T, N_1))$ and satisfies $\frac{df^B_t(S)}{dS} = 0$ for $T_1 < T < T^D$, and $S^B_t = \min(T, N_1)$ for $T^D \leq T < N_1 + N_2$. On the other hand, $f^U_t(S)$ is a linear function of S with slope equal to $\mathrm{E}[(t+\alpha)1_{t+\alpha \leq p^U_{2t}}]$, which is positive when $p^U_{2t} > t + \underline{\alpha}$ (i.e., when $t < \bar{t}$) and is zero otherwise (i.e., when $t \geq \bar{t}$). That is, $f^U_t(S)$ strictly increases in S when $t < \bar{t}$ and is independent of S when $t \geq \bar{t}$.

Clearly the location of S^f_t depends on the values of T and t. Below we consider three cases:

- $0 < T < N_1 + N_2$: It suffices to show $S^f_t = S^B_t$. By equation (S.1), it then suffices to show that when $S^=_t \in [0, \min(T, N_1)]$ (implying that $\pi^f_t(S) = f^U_t(S)$ for some $S \in [0, \min(T, N_1)]$), $f^U_t(S)$ strictly increases in S. Prove by contradiction. Suppose that $f^U_t(S)$ is independent of S. This occurs only if $t \geq \bar{t}$ and $\overline{G}(p^U_{2t} - t) = 1$. This, however, implies that $\frac{T-S}{N_1+N_2-S} < \overline{G}(p^U_{2t} - t)$ for all $S \in [0, \min(T, N_1)]$, and contradicts with the assumption $S^=_t \in [0, \min(T, N_1)]$.

Also, note a special case: when $T_1 < T < N_1 + N_2$, since $f^U_t(S)$ strictly increases and $f^B_t(S)$ is strictly quasi-concave, clearly $\pi^f_t(S)$ strictly increases in $S \in [0, S^f_t)$. This is used in proof of Theorem 3.

- $T \geq N_1 + N_2$ and $t < \bar{t}$: In such a case, $T - S \geq (N_1 + N_2 - S)\overline{G}(p^U_{2t} - t)$ and $\pi^f_t(S) = f^U_t(S)$ for all $S \in [0, \min(T, N+1)]$. Since $f^U_t(S)$ strictly increases in S, so is $\pi^f_t(S)$ and thus, $S^f_t = \min(T, N_1)$.
- $T \geq N_1 + N_2$ and $t \geq \bar{t}$: Similarly to the second bullet, $\pi^f_t(S) = f^U_t(S)$ for all $S \in [0, \min(T, N+1)]$. Since $f^U_t(S)$ is independent of S for $t \geq \bar{t}$, so is $\pi^f_t(S)$ and thus, S^f_t can be any value between zero and $\min(T, N_1)$. □

Section SB Proofs of All Technical Lemmas

LEMMA C.1 $p^*_{2H}(S) \geq p^*_{2L}(S)$, $p^*_{1H}(S) > p^*_{1L}(S)$.

Proof of Lemma C.1 **To show** $p^*_{2H}(S) \geq p^*_{2L}(S)$, since $p^*_{2t}(S) = \max(p^U_{2t}, p^B_{2t}(S))$ (ref. Lemma 1), it suffices to show that both p^U_{2t} and $p^B_{2t}(S)$ increase in t. The monotonicity of $p^B_{2t}(S)$ immediately follows from its definition: $p^B_{2t}(S) = t + (\overline{G})^{-1}\left(\frac{T-S}{N_1+N_2-S}\right)$. For p^U_{2t}, the monotonicity follows from its expression in equation (3) and the fact that $\frac{\overline{G}(p_2-t)}{g(p_2-t)}$ increases in t (due to IFR property of $G(\cdot)$).

To show $p^*_{1H}(S) > p^*_{1L}(S)$, it suffices to show that $p^*_{1t}(S)$ strictly increases in t. Recall that $p^*_{1t}(S) = \mathrm{E}_\alpha[\min(p^*_{2t}(S), t+\alpha)]$, where $p^*_{2t}(S) \in [t+\underline{\alpha}, t+\overline{\alpha}]$ and is continuous in t (ref. Lemma 1). The monotonicity of $p^*_{1t}(S)$ is trivial when $p^*_{2t}(S) = t+\underline{\alpha}$ or $t+\overline{\alpha}$. Now, if $p^*_{2t}(S) \in (t+\underline{\alpha}, t+\overline{\alpha})$, by envelop theorem, $\frac{dp^*_{1t}(S)}{dt} = \frac{\partial p^*_{1t}(S)}{\partial p^*_{2t}} \cdot \frac{dp^*_{2t}(S)}{dt} + \frac{\partial p^*_{1t}(S)}{\partial t} = \overline{G}(p^*_{2t}(S)-t) \cdot \frac{dp^*_{2t}(S)}{dt} + G(p^*_{2t}(S)-t)$, which is positive by the facts that $p^*_{2t}(S)$ increases in t (proved above) and $G(p^*_{2t}(S)-t) > 0$. □

LEMMA D.1 $\pi^*_{2H}(S) - \pi^*_{2L}(S)$ *strictly decreases in* S.

Proof of Lemma D.1 We first prove that $\overline{G}(p^U_{2H}-H) \geq \overline{G}(p^U_{2L}-L)$ and $p^U_{2H}\overline{G}(p^U_{2H}-H) > p^U_{2L}\overline{G}(p^U_{2L}-L)$, and then use these results to show the lemma.

Let $\Delta_1 = \overline{G}(p^U_{2H}-H) - \overline{G}(p^U_{2L}-L)$ and $\Delta_2 = p^U_{2H}\overline{G}(p^U_{2H}-H) - p^U_{2L}\overline{G}(p^U_{2L}-L)$. By equation (3), it suffices to consider the following three cases:

- $L \geq \bar{t}$: $p^U_{2H} = H + \underline{\alpha}$ and $p^U_{2L} = L + \underline{\alpha}$. Hence, $\Delta_1 = 1 - 1 = 0$ and $\Delta_2 = p^U_{2H} - p^U_{2L} = H - L > 0$.
- $L < \bar{t} \leq H$: $p^U_{2H} = H + \underline{\alpha}$ and $p^U_{2L} \in (L+\underline{\alpha}, L+\overline{\alpha})$. Hence, $\Delta_1 = 1 - \overline{G}(p^U_{2L}-L) > 0$, and by Lemma C.1, $\Delta_2 = p^U_{2H} - p^U_{2L}\overline{G}(p^U_{2L}-L) \geq p^U_{2L} - p^U_{2L}\overline{G}(p^U_{2L}-L) = p^U_{2L}G(p^U_{2L}-L) > 0$.
- $H < \bar{t}$: $p^U_{2t} \in (t+\underline{\alpha}, t+\overline{\alpha})$ for $t = H$ and L. It suffices to show that both $\overline{G}(p^U_{2t}-t)$ and $p^U_{2t}\overline{G}(p^U_{2t}-t)$ strictly increase in t. By equation (3) and IFR property of $G(\cdot)$, $d\overline{G}(p^U_{2t}-t)/dt = \{g(x)[1-(\frac{\overline{G}(x)}{g(x)})']^{-1}\}|_{x=p^U_{2t}-t} > 0$ and $d\{p^U_{2t}\overline{G}(p^U_{2t}-t)\}/dt = \overline{G}(p^U_{2t}-t) > 0$.

Now, to show $\pi^*_{2H}(S) - \pi^*_{2L}(S)$ strictly decreases in S, recall that for $S \in [0, \min(T, N_1)]$,

$$\pi^*_{2t}(S) = \pi_{2t}(\max(p^U_{2t}, p^B_{2t}(S)), S) = \begin{cases} (N_1+N_2-S)p^U_{2t}\overline{G}(p^U_{2t}-t) & \text{if } \frac{T-S}{N_1+N_2-S} \geq \overline{G}(p^U_{2t}-t) \\ (T-S)p^B_{2t}(S) & \text{if } \frac{T-S}{N_1+N_2-S} \leq \overline{G}(p^U_{2t}-t) \end{cases}$$

where p^U_{2t} is independent of S and $p^B_{2t}(S) = (\overline{G})^{-1}\left[\min\left(\frac{T-S}{N_1+N_2-S}, 1\right)\right]$.

To show that $\pi^*_{2H}(S) - \pi^*_{2L}(S)$ strictly decreases in S, by the fact $\overline{G}(p^U_{2H}-H) \geq \overline{G}(p^U_{2L}-L)$, it suffices to consider the following three cases:

- $\frac{T-S}{N_1+N_2-S} \geq \overline{G}(p^U_{2H}-H)$:

In such a case, $p^*_{2H}(S) = p^U_{2H}$, $p^*_{2L}(S) = p^U_{2L}$, and

$$\pi^*_{2H}(S) - \pi^*_{2L}(S) = (N_1+N_2-S)\left[p^U_{2H}\overline{G}(p^U_{2H}-H) - p^U_{2L}\overline{G}(p^U_{2L}-L)\right],$$

which is linear and strictly decreases in S by part (i).

- $\overline{G}(p^U_{2L}-L) \leq \frac{T-S}{N_1+N_2-S} \leq \overline{G}(p^U_{2H}-H)$:

In such a case, $p^*_{2H}(S) = p^B_{2H}(S)$, $p^*_{2L}(S) = p^U_{2L}$, and

$$\pi^*_{2H}(S) - \pi^*_{2L}(S) = p^B_{2H}(S)(T-S) - (N_1+N_2-S)p^U_{2L}\overline{G}(p^U_{2L}-L) \tag{S.2}$$

Taking derivative of equation (S.2) with respect to S:

$$\frac{d\pi_{2H}^*(S)}{dS} - \frac{d\pi_{2L}^*(S)}{dS} = (T-S)\frac{dp_{2H}^B(S)}{dS} - p_{2H}^B(S) + p_{2L}^U\overline{G}(p_{2L}^U - L) \tag{S.3}$$

Note that since $\frac{T-S}{N_1+N_2-S} \le \overline{G}(p_{2H}^U - H) \le 1$, $p_{2H}^B(S) = (\overline{G})^{-1}\left[\frac{T-S}{N_1+N_2-S}\right]$. Taking derivative of $p_{2H}^B(S)$ with respect to S, we get $\frac{dp_{2H}^B(S)}{dS} = \frac{N_1+N_2-T}{(N_1+N_2-S)^2\cdot g(p_{2H}^B(S)-H)}$. Substituting it into equation (S.3) and noting that $G(p_{2H}^B(S)-H) = \frac{N_1+N_2-T}{N_1+N_2-S}$ and $\overline{G}(p_{2H}^B(S)-H) = \frac{T-S}{N_1+N_2-S}$:

$$\begin{aligned}\frac{d\pi_{2H}^*(S)}{dS} - \frac{d\pi_{2L}^*(S)}{dS} &= \frac{G(p_{2H}^B(S)-H)\overline{G}(p_{2H}^B(S)-H)}{g(p_{2H}^B(S)-H)} - p_{2H}^B(S) + p_{2L}^U\overline{G}(p_{2L}^U - L)\\ &= \left[\frac{\overline{G}(p_{2H}^B(S)-H)G(p_{2H}^B(S)-H)}{g(p_{2H}^B(S)-H)} - (p_{2H}^B(S)-H)\right] - H + p_{2L}^U\overline{G}(p_{2L}^U - L)\end{aligned} \tag{S.4}$$

Note that by IFR property of $G(\cdot)$, $\left(\frac{\overline{G}(x)G(x)}{g(x)} - x\right)' = \left(\frac{\overline{G}(x)}{g(x)}\right)' G(x) - G(x) \le 0$. Meanwhile, since $\frac{T-S}{N_1+N_2-S} \le \overline{G}(p_{2H}^U - H)$, $p_{2H}^B(S) \ge p_{2H}^U$. Furthermore, by equation (3), $p_{2H}^U \ge \frac{\overline{G}(p_{2H}^U-H)}{g(p_{2H}^U-H)}$ (the inequality is strict for $t > \bar{t}$ and holds as equality for $t \le \bar{t}$). Apply these facts and part (i) to equation (S.4):

$$\begin{aligned}\frac{d\pi_{2H}^*(S)}{dS} - \frac{d\pi_{2L}^*(S)}{dS} &\le \left[\frac{\overline{G}(p_{2H}^U-H)G(p_{2H}^U-H)}{g(p_{2H}^U-H)} - (p_{2H}^U - H)\right] - H + p_{2L}^U\overline{G}(p_{2L}^U - L)\\ &= \frac{\overline{G}(p_{2H}^U-H)G(p_{2H}^U-H)}{g(p_{2H}^U-H)} - p_{2H}^U + p_{2L}^U\overline{G}(p_{2L}^U - L)\\ &\le p_{2H}^U G(p_{2H}^U - H) - p_{2H}^U + p_{2L}^U\overline{G}(p_{2L}^U - L)\\ &= -p_{2H}^U\overline{G}(p_{2H}^U - H) + p_{2L}^U\overline{G}(p_{2L}^U - L) < 0.\end{aligned}$$

- $\frac{T-S}{N_1+N_2-S} \le \overline{G}(p_{2L}^U - L)$:

In such a case, $\frac{T-S}{N_1+N_2-S} \le 1$ and $p_{2t}^*(S) = p_{2t}^B(S) = t + (\overline{G})^{-1}\left(\frac{T-S}{N_1+N_2-S}\right)$ for $t = H$ and L, and

$$\pi_{2H}^*(S) - \pi_{2L}^*(S) = \left(p_{2H}^B(S) - c\right)(T-S) - \left(p_{2L}^B(S) - c\right)(T-S) = (H-L)(T-S),$$

which strictly decreases in S since $H > L$. □

LEMMA H.1 *When capacity rationing is not allowed and customers in advance are perfectly informed of quality, there exists a function $t^D(T)$ for $T > 0$ such that if $t \le t^D(T)$, the seller should offer full advance selling; otherwise, the seller should sell only in spot.*

Proof of Lemma H.1 First note that by Theorem 1, for all t, the seller should offer full advance selling when $T \le T_1$ and should sell only in spot when $T \ge T_D$. Equivalently, let $t^D(T) = -\infty$ for $T \le T_1$ and $t^D(T) = \infty$ for $T \ge T_D$. For $T \in (T_1, T_D)$, define the profit difference $\pi_t^f(\min(T, N_1)) - \pi_t^f(0)$ as a function of t: $\Delta(t) = \pi_t^f(\min(T, N_1)) - \pi_t^f(0)$. To show the existence of $t^D(T)$, it suffices to show that (i) when $T \in (T_1, T_D)$, $T < N_1 + N_2\overline{G}(p_{2t}^U - t)$ for all t; (ii) for t satisfying $T < (N_1 + N_2)\overline{G}(p_{2t}^U - t)$ (i.e., t sufficiently large), $\Delta(t)$ is independent of t; (iii) for t satisfying $(N_1 + N_2)\overline{G}(p_{2t}^U - t) \le T < N_1 + N_2\overline{G}(p_{2t}^U - t)$ (i.e., t sufficiently small), $\Delta(t)$ is nondecreasing in t.

(i) Prove by contradiction. Suppose $T \ge N_1 + N_2\overline{G}(p_{2t}^U - t)$. In such a case, $T - S \ge (N_1 + N_2 - S)\overline{G}(p_{2t}^U - t)$ for all $S \in [0, \min(T, N_1)]$, implying $p_{2t}^*(S) = p_{2t}^U$ for all S. Applying this to the expression of $\pi_t^f(S)$ in equation (7): $\pi_t^f(S) = \{\mathrm{E}[\min(t+\alpha, p_{2t}^U)] - p_{2t}^U\overline{G}(p_{2t}^U - t)\}S + (N_1 + N_2)p_{2t}^U\overline{G}(p_{2t}^U - t)$. Since $\mathrm{E}[\min(t+\alpha, p_{2t}^U)] - p_{2t}^U\overline{G}(p_{2t}^U - t) = \mathrm{E}[(t+\alpha)1_{t+\alpha < p_{2t}^U}] \ge 0$, $\pi_t^f(S)$ is linear and increasing in S. Hence, $S_t^f = \min(T, N_1)$. However, this contradicts with Theorem 1 and the fact $T \in (T_1, T_D)$.

(ii) If $T < (N_1+N_2)\overline{G}(p_{2t}^U - t)$, $T - S < (N_1+N_2-S)\overline{G}(p_{2t}^U - t)$ for all $S \in [0, \min(T, N_1)]$, implying $p_{2t}^*(S) = p_{2t}^B(S) = t + (\overline{G})^{-1}(\frac{T-S}{N_1+N_2-S})$ for all S. Applying this to expression of $\Delta(t)$:

$$\begin{aligned}\Delta(t) =& \mathrm{E}[\min(t+\alpha, p_{2t}^B(\min(T,N_1)))]\min(T,N_1) + (T-\min(T,N_1))p_{2t}^B(\min(T,N_1)) - Tp_{2t}^B(0)\\ =& \mathrm{E}[\min(\alpha, (\overline{G})^{-1}(\tfrac{T-\min(T,N_1)}{N_1+N_2-\min(T,N_1)}))]\min(T,N_1) + (T-\min(T,N_1))(\overline{G})^{-1}(\tfrac{T-\min(T,N_1)}{N_1+N_2-\min(T,N_1)}) - T(\overline{G})^{-1}(\tfrac{T}{N_1+N_2})\end{aligned}$$

Clearly, $\Delta(t)$ is independent of t.

(iii) If $(N_1+N_2)\overline{G}(p_{2t}^U - t) \le T < N_1 + N_2\overline{G}(p_{2t}^U - t)$, $p_{2t}^*(0) = p_{2t}^U$ and $p_{2t}^*(\min(T,N_1)) = p_{2t}^B(\min(T,N_1))$. Meanwhile, it is implied that $\overline{G}(p_{2t}^U - t) < 1$, i.e., $t < \bar{t}$. By the definition of $\pi_t^f(S)$,

$$\Delta(t) = \mathrm{E}[\min(t+\alpha, p_{2t}^B(\min(T,N_1)))]\min(T,N_1) + (T-\min(T,N_1))p_{2t}^B(\min(T,N_1)) - (N_1+N_2)p_{2t}^U\overline{G}(p_{2t}^U - t)$$

Recall that by equation (3), for $t < \bar{t}$, p_{2t}^U is the maximizer of $p_2\overline{G}(p_2 - t)$ and satisfies $p_2 = \frac{\overline{G}(p_2-t)}{g(p_2-t)}$. By envelop theorem, $\frac{\partial\Delta(t)}{\partial t} = T - (N_1+N_2)p_{2t}^U g(p_{2t}^U - t) = T - (N_1+N_2)\overline{G}(p_{2t}^U - t) \ge 0$. □

Lemma H.2 *When capacity rationing is not allowed, in any separating equilibrium, L type offers full advance selling at price* $p_1^L(\min(T,N_1))$*, while H type sells only in spot.*

Proof of Lemma H.2 It suffices to show that (i) no separating equilibrium exists where both types offer full advance selling; (ii) no separating equilibrium exists where H type offers full advance selling and L type sells only in spot; (iii) L type's equilibrium price is $p_1^L(\min(T,N_1))$.

(i) Suppose there exists a separating equilibrium where both types of sellers offer full advance selling with different advance prices $p_{1H} \neq p_{1L}$. Without loss of generality, assume $p_{1H} > p_{1L}$. By definition of a participating equilibrium, all customers buy in advance upon observing p_{1H} or p_{1L}. Such separating equilibrium, however, cannot be sustained since L type always has an incentive to mimic H type and charge p_{1H} in advance, which strictly increases his advance profit and does not influence his spot profit.

(ii) Suppose there exists a separating equilibrium such that H type offers advance selling at p_{1H} and L type sells only in spot. By Lemma D.1,

$$\pi_L^a(p_{1H}, \min(T,N_1), 1) - \pi_{2L}^*(0) > \pi_H^a(p_{1H}, \min(T,N_1), 1) - \pi_{2H}^*(0) \tag{S.5}$$

Equation (S.5) implies that if advance selling at p_{1H} is beneficial for H type, it is also beneficial for L type. Hence, L type always has an incentive to mimic H type and the separating equilibrium cannot be sustained.

(iii) L type's equilibrium price equals to $p_1^L(\min(T,N_1))$ because any price higher than $p_1^L(\min(T,N_1))$ will be rejected by customers, and any price lower than $p_1^L(\min(T,N_1))$ is strictly dominated by it. □

Lemma H.3 *(i) In the focal pooling equilibrium where both types of sellers sell in advance, the equilibrium price is* $p_1^E = qp_{1H}^*(\min(T,N_1)) + (1-q)p_{1L}^*(\min(T,N_1))$*. (ii) The focal pooling equilibrium is sustained only if* $H \le t^D(T)$*. (iii) When* $H \le t^D(T)$*, there exist a threshold* $\bar{\delta} \ge 0$ *and a function* $\bar{q}(\delta) \in [0,1]$ *for* $\delta > 0$ *such that the focal pooling equilibrium is sustained if either* $\delta \le \bar{\delta}$ *or* $q \ge \bar{q}(\delta)$*.*

Proof of Lemma H.3 (i) To show that the equilibrium price is p_1^E, note that in a pooling equilibrium, the posterior belief is always the same as the prior belief, q. By equation (5) and the fact $S = \min(T, N_1)$ in a no-rationing equilibrium, customers' willingness to pay in advance is p_1^E as defined. Since customers buy in advance in the pooling equilibrium (ref. participating equilibrium), the pooling price cannot exceed p_1^E. On the other hand, all pooling equilibria with advance price less than p_1^E are pareto dominated from the seller's point of view. Therefore, the equilibrium price is p_1^E in any focal pooling equilibrium.
(ii) Note that the pooling equilibrium is sustained if and only if offering advance selling at price p_1^E, compared to selling only in spot, makes both types of sellers better off. That is,

$$p_1^E \min(T, N_1) + \pi_{2L}^*(\min(T, N_1)) - \pi_{2L}^*(0) \geq 0 \tag{S.6}$$

$$p_1^E \min(T, N_1) + \pi_{2H}^*(\min(T, N_1)) - \pi_{2H}^*(0) \geq 0 \tag{S.7}$$

By Lemma D.1, equation (S.7) implies equation (S.6). Hence, the pooling equilibrium is sustained if and only if equation (S.7) holds. By Lemma C.1 and the fact $q \in [0,1]$, $p_1^E < p_{1H}^*(\min(T, N_1))$. Hence, if equation (S.7) holds, $p_{1H}^*(\min(T, N_1)) \min(T, N_1) + \pi_{2H}^*(\min(T, N_1)) \geq \pi_{2H}^*(0)$, further implying $H \leq t^D(T)$ by Lemma H.1.
(iii) In preparation, substitute $L = H - \delta$ and expression of p_1^E to equation (S.7) and get:

$$[q p_{1H}^*(\min(T, N_1)) + (1-q) p_{1\{H-\delta\}}^*(\min(T, N_1))] \min(T, N_1) + \pi_{2H}^*(\min(T, N_1)) \geq \pi_{2H}^*(0) \tag{S.8}$$

To see when equation (S.8) holds, define a function of δ: $f(\delta) = p_{1\{H-\delta\}}^*(\min(T, N_1)) \min(T, N_1) + \pi_{2H}^*(\min(T, N_1)) - \pi_{2H}^*(0)$. Since $p_{1\{H-\delta\}}^*(\min(T, N_1))$ strictly decreases in δ (Lemma C.1), so does $f(\delta)$. Furthermore, $f(0) \geq 0$ by the fact $H \leq t^D(T)$ and Lemma H.1. Thus, there exists $\bar{\delta} \geq 0$ satisfying $f(\delta) = 0$. If $\delta \leq \bar{\delta}$, $f(\delta) \geq 0$ and equation (S.8) holds for all q. In such a case, let $\bar{q}(\delta) = 0$. If, however, $\delta > \bar{\delta}$, $f(\delta) < 0$. For given δ, since $q p_{1H}^*(\min(T, N_1)) + (1-q) p_{1\{H-\delta\}}^*(\min(T, N_1))$ increases in q, there exists $\bar{q}(\delta) \in [0,1]$ such that equation (S.8) holds if and only if $q \geq \bar{q}(\delta)$. □

Section SC Three Extensions

SC.1. Signaling: Rationing versus Advertising

So far we have shown that a seller can use capacity rationing to signal product quality during the advance sales. We now elaborate how capacity rationing differs from advertising, another signaling tool that has been studied extensively in the literature. Although advertisement can help the seller in many different ways (e.g., raising the willingness to pay, increasing the market size), we consider a pure signaling role of advertisement. Thus, we assume that advertising affects neither valuation distribution nor market size. Instead, we consider the case where advertising is a pure dissipative cost that the seller incurs to its customers for signaling. Such uninformative advertising has been considered in the literature, see Kihlstrom and Riordan (1984), Milgrom and Roberts (1986), Bagwell and Ramey (1988), Stock and Balachander (2005), and the references therein.

If customers were perfectly informed about product quality in advance, advertising would not be used in our settings. With unknown quality, advertising may enable the seller to convince customers about quality of the product and charge a higher price during the advance sales. Clearly, there is no benefit of advertising if the seller sells only in the spot period, when quality is already revealed.

To evaluate effectiveness of advertising in our setting, consider a case when seller uses adverting in the advance period instead of rationing. That is, S must be either zero (no advance sales) or $\min(T, N_1)$, and advertising may be used in advance. All other settings, including the sequence of events, are the same as before. In the first stage, the seller decides whether to offer advance selling or not, i.e., $S=0$ or $\min(T,N_1)$. If he does ($S=\min(T,N_1)$), then the seller chooses the advance price p_1 and the advertising expenditure A. Having observed the price and advertising expense, customers form a posterior belief about the product being high quality, denoted by $b(p_1,S,A)$. For given posterior belief, the maximum price that customers will accept during the advance period is $b(p_1,S,A)p_{1H}^*(S)+(1-b(p_1,S,A))p_{1L}^*(S)$. Hence, the seller offers an advance selling if $p_1 \le b(p_1,S,A)p_{1H}^*(S)+(1-b(p_1,S,A))p_{1L}^*(S)$. The seller's expected profit over the two periods is

$$\pi_t^{a,AD}(p_1,S,A,b)=p_1S+\pi_{2t}^*(S)-A.$$

Otherwise, the seller's profit is $\pi_t^{a,AD}(p_1,S,A,b)=\pi_{2t}^*(0)$. The next result characterizes the equilibrium.

Theorem 7 *(i) If $H \le t^D(T)$ and $\delta \le \overline{\delta}$, there exists a separating equilibrium under which both types offer full advance selling, but only the high-type seller advertises. In all other cases, no seller advertises.*
(ii) The separating equilibrium in part (i) is Pareto-dominated by a no-advertising pooling equilibrium.

Under the separating equilibrium described in part (i), the high-type seller proves his type through advertising that the low-type seller cannot afford. This equilibrium can be sustained only when both types find it optimal to sell in advance under the full information case ($H \le t^D(T)$) and the quality difference is sufficiently small ($\delta \le \overline{\delta}$). The first condition is straight-forward because, if a seller prefers spot-only selling under the full information case, his preference would remain the same in the asymmetric information case, since advertising only decreases the seller's profit from advance selling. For the second condition, notice that the difference in the advance prices between the two types of seller will be large when the quality difference is large. Consequently, it is very attractive for the low-type to mimic the high-type. To separate from the low-type, the high-type needs to incur a high advertising cost, which makes advance selling less appealing. When the quality difference is sufficiently large, the high-type is better off selling only in spot.

Even when the separating equilibrium in part (i) can be sustained, the high type has to spend significantly on advertising. In fact, the expected profit that the high-type seller earns from sales in the advance period will be the same as that of the low-type seller. To see why, first recall that since the true quality will be revealed in spot, the advertisement intends to influence the advance sales and profit. In the separating equilibrium where both offer full advance selling, although the high type charges a higher advance price than the low type, any extra revenue that the high-type earns will be fully offset by the advertising cost - otherwise it will incentivize the low type to mimic. Under this situation, there is another equilibrium that is better for both types of seller: pooling with no advertisement. The low type earns more than under the separating equilibrium since the pooling price is higher than her separating price, and the high-type is better off pooling as he does not incur advertising cost. It can be shown that this pooling equilibrium Pareto dominates the separating equilibrium. Obviously, our analysis assumes a very limited role of advertising. Advertising may increase the valuation distribution or market size, factors we do not study in this paper.

Now, consider the case that the seller is allowed to ration, i.e., S can be any value between zero and $\min(T,N_1)$. Applying similar analysis, it can be shown that the intuitive criterion used in Theorem 4 rules out any pooling equilibrium in which both types sell in advance. Also, in any separating equilibrium, only the high-type seller will signal its quality as the low-type seller will follow his full-information strategy $\left(p^f_{1L}, S^f_L, 0\right)$ and place no advertisement. It turns out that the high-type seller prefers to use rationing over advertising as a signal. The result is summarized below.

Theorem 8 *Neither of the two sellers invests in advertising in any separating equilibrium.*

Theorem 8 shows that signaling by rationing is more efficient than by advertising. To understand this, compare these two signaling levers. Advertising is a pure cost to the seller: in other words, the seller will not advertise if the product quality is publicly known. On the other hand, advance selling (and capacity rationing) can increase seller's profit even when there is no quality uncertainty (Theorem 1). Allowing the seller to choose its rationing quantity helps the seller (at least partially) offset the signaling cost, as the seller can adjust availability and price in the spot period according to the cost of signal.

Proof of Theorem 7 We follow three steps to prove the result: (i-a) positive advertising expenditure can be sustained only in a separating equilibrium where both types sell in advance and only high type advertises; (i-b) the separating equilibrium in point (i-a) is sustained if and only if $L < H \leq t^D(T)$ and $\delta \leq \overline{\delta}$; (ii) whenever the separating equilibrium in point (i-a) is sustained, it is pareto dominated by a focal pooling equilibrium where both types sell in advance and neither advertises.

(i-a) First, similarly to the proof of Lemma H.2, it can be shown that no separating equilibrium exists where only high type sells in advance. Likewise, no separating equilibrium exists where both types sell in advance and advertise, or only low type does so. In either case, the low type would be better off deviating to no advertising at all, since advertising can neither increase sales nor improve margin for her. Furthermore, any pooling equilibrium where both types sell and advertise in advance is pareto dominated by the focal pooling equilibrium where both sell in advance and yet neither advertises. Thus, the only possible scenario to sustain a positive advertising spending is a separating equilibrium where both types offer advance selling and yet only high type advertises.

(i-b) A separating equilibrium where both types sell in advance and only high type advertises exists if and only if there exists some $A > 0$ satisfying the following four conditions:

$$p^*_{1H}(\min(T,N_1))\min(T,N_1) + \pi^*_{2H}(\min(T,N_1)) - A \geq \pi^*_{2H}(0)$$
$$p^*_{1L}(\min(T,N_1))\min(T,N_1) + \pi^*_{2L}(\min(T,N_1)) \geq \pi^*_{2L}(0)$$
$$p^*_{1H}(\min(T,N_1))\min(T,N_1) + \pi^*_{2L}(\min(T,N_1)) - A \leq p^*_{1L}(\min(T,N_1))\min(T,N_1) + \pi^*_{2L}(\min(T,N_1))$$
$$p^*_{1H}(\min(T,N_1))\min(T,N_1) + \pi^*_{2H}(\min(T,N_1)) - A \geq p^*_{1L}(\min(T,N_1))\min(T,N_1) + \pi^*_{2H}(\min(T,N_1))$$

where the first two inequalities ensure that both types prefer selling in advance to selling only in spot, and the last two inequalities guarantee that neither type has an incentive to mimic the other type. After simplifying, these conditions are equivalent to

$$p^*_{1H}(\min(T,N_1))\min(T,N_1) + \pi^*_{2H}(\min(T,N_1)) - \pi^*_{2H}(0) \geq A = [p^*_{1H}(\min(T,N_1)) - p^*_{1L}(\min(T,N_1))]\min(T,N_1)$$
$$p^*_{1L}(\min(T,N_1))\min(T,N_1) + \pi^*_{2L}(\min(T,N_1)) \geq \pi^*_{2L}(0)$$

By the proof of Lemma H.3, these conditions are satisfied if and only if $H \leq t^D(T)$ and $\delta \leq \overline{\delta}$.

(ii) From the equilibrium conditions in point (i-b), it is easy to see that the type-t seller's profit in the separating equilibrium equals to $p^*_{1L}(\min(T,N_1))\min(T,N_1) + \pi^*_{2t}(\min(T,N_1))$. However, when $H \leq t^D(T)$ and $\delta \leq \overline{\delta}$, by

Lemma H.3, a no-advertising pooling equilibrium also exists, where the type-t seller's equilibrium profit equals to $p_1^E \min(T,N_1)+\pi_{2t}^*(\min(T,N_1))$. Since $p_1^E \geq p_{1L}^*(\min(T,N_1))$, the separating equilibrium is pareto dominated by the pooling equilibrium.

Proof of Theorem 8 First note that L type never invests in advertising in any separating equilibrium, where he always follows his full-information strategy. Denote H type's equilibrium strategy by (p_1^*, S^*, A^*). Per definition of a separating equilibrium, (p_1^*, S^*, A^*) is a solution to the following problem:

$$\max_{p_1,S,Q} \pi_H^{a,AD}(p_1,S,Q,1)=p_1 S+\pi_{2H}^*(S)-Q$$

$$\text{subject to} \quad S=0 \text{ or } S\in(0,\min(T,N_1)] \text{ and } p_1\in[p_{1L}^*(S),p_{1H}^*(S)], Q\geq 0 \tag{S.9}$$

$$\pi_L^{a,AD}(p_1,S,Q,1)\leq \pi_L^{a,AD}\left(p_{1L}^f,S_L^f,0,0\right) \tag{S.10}$$

$$\pi_H^{a,AD}(p_1,S,Q,1)\geq \pi_H^{a,AD}\left(p_{1L}^f,S_L^f,0,0\right) \tag{S.11}$$

$$(p_1,S,Q)\neq\left(p_{1L}^f,S_L^f,0\right) \tag{S.12}$$

We prove $A^*=0$ by contradiction. Suppose $A^*>0$. To reach a contradiction, it suffices to show that there exists a feasible strategy $(p_1', S', 0)$ which strictly improves H type's profit from what he can get by following strategy (p_1^*, S^*, A^*).

To this end, first note that when $A^*>0$, H type should sell in advance in equilibrium (i.e., $S^*>0$), since otherwise he could not enjoy any benefit from the advertising. Meanwhile, constraint (S.10) must hold as equality at (p_1^*, S^*, A^*), since otherwise A^* can be decreased by a small amount such that all the constraints are satisfied and H type's total profit is strictly improved. That is,

$$\pi_L^{a,AD}\left(p_1^*,S^*,A^*,1\right)=\pi_L^{a,AD}\left(p_{1L}^f,S_L^f,0,0\right). \tag{S.13}$$

Subtracting equation (S.13) from equation (S.11), we get

$$\pi_H^{a,AD}\left(p_1^*,S^*,A^*,1\right)-\pi_L^{a,AD}\left(p_1^*,S^*,A^*,1\right)\geq \pi_H^{a,AD}\left(p_{1L}^f,S_L^f,0,0\right)-\pi_L^{a,AD}\left(p_{1L}^f,S_L^f,0,0\right),$$

which further implies,

$$\pi_{2H}^*(S^*)-\pi_{2L}^*(S^*)\geq \pi_{2H}^*(S_L^f)-\pi_{2L}^*(S_L^f) \tag{S.14}$$

By Lemma D.1, equation (S.14) implies $S_L^f\geq S^*>0$.

Now, define a function of S for $S\in[0,\min(T,N_1)]$: $M(S)=\pi_L^{a,AD}\left(p_{1H}^*(S),S,0,1\right)-\pi_L^{a,AD}\left(p_{1L}^f,S_L^f,0,0\right)$. Clearly $M(S)$ is continuous in S. Furthermore, $M(0)\leq 0$ since $\left(p_{1L}^f,S_L^f,0\right)$ is L type's full-information strategy. Meanwhile, by equations (S.9) and (S.13),

$$\begin{aligned}
M(S^*)&=\pi_L^{a,AD}\left(p_{1H}^*(S^*),S^*,0,1\right)-\pi_L^{a,AD}\left(p_{1L}^f,S_L^f,0,0\right)\\
&\geq \pi_L^{a,AD}\left(p_1^*,S^*,0,1\right)-\pi_L^{a,AD}\left(p_{1L}^f,S_L^f,0,0\right)\\
&=\pi_L^{a,AD}\left(p_1^*,S^*,A^*,1\right)+A^*-\pi_L^{a,AD}\left(p_{1L}^f,S_L^f,0,0\right))=A^*>0
\end{aligned}$$

Hence, there exists a $S'\in[0,S^*)$ such that $M(S')=0$. That is,

$$\pi_L^{a,AD}\left(p_{1H}^*(S'),S',0,1\right)=\pi_L^{a,AD}\left(p_{1L}^f,S_L^f,0,0\right)=\pi_L^{a,AD}\left(p_1^*,S^*,A^*,1\right) \tag{S.15}$$

Since $S' < S^* \le S_L^f$, it is easy to check that $(p_{1H}^*(S'), S', 0)$ satisfies all constraints. Furthermore, noting $S' < S^*$, by equation (S.15) and Lemma D.1, we have

$$\begin{aligned}
&\pi_H^{a,AD}(p_{1H}^*(S'), S', 0, 1) - \pi_H^{a,AD}(p_1^*, S^*, A^*, 1)\\
&= \pi_L^{a,AD}(p_{1H}^*(S'), S', 0, 1) + \pi_{2H}^*(S') - \pi_{2L}^*(S') - \left[\pi_L^{a,AD}(p_1^*, S^*, A^*, 1) + \pi_{2H}^*(S^*) - \pi_{2L}^*(S^*)\right]\\
&= \pi_{2H}^*(S') - \pi_{2L}^*(S') - [\pi_{2H}^*(S^*) - \pi_{2L}^*(S^*)] > 0
\end{aligned}$$

That is, compared to (p_1^*, S^*, A^*), $(p_{1H}^*(S'), S', 0)$ strictly improves H type's profit. This is a contradiction with the optimality of (p_1^*, S^*, A^*). Hence, $A^* = 0$.

SC.2. Quality Adjustment Cost

Our base model assumes that all sellers incur the same marginal cost, which is normalized to zero. In some situations, producing a high quality product incurs higher marginal cost. To examine this scenario, we generalize the basic model and assume that the high-quality seller incurs a higher marginal cost than the low-quality seller: $c_H > c_L = 0$. We assume that the increase in customer's utility from high quality is higher than the cost of high quality, i.e., $H - L > c_H$. The following theorem shows that all the results continue to hold.

Theorem 9 *Suppose that the marginal cost of the high-quality seller, c_H, is higher than that of the low-quality seller and satisfies $0 = c_L < c_H < H - L$. Then, all the results in the original model, Theorem 1 through Theorem 8, hold, with the correction on the marginal cost of the high-type seller.*

Proof of Theorem 9 To explicitly recognize the dependence of the high-type seller's strategies and profits on the marginal cost, within this proof we write all of them as functions of c, e.g., $p_{2t}^*(c, S)$, $S_t^f(c)$, and $\pi_{2t}^*(c, S)$. Let $H' = H - c_H > L$. We will first show that under the full-information case, $p_{2H}^*(c_H, S) = p_{2H'}^*(0, S) + c_H$, $p_{1H}^*(c_H, S) = p_{1H'}^*(0, S) + c_H$, $\pi_{2H}^*(c_H, S) = \pi_{2H'}^*(0, S)$, and $S_H^f(c_H) = S_{H'}^f(0)$, and then use these results to prove all the lemmas and theorems.

Full Information

For $p_{2H}^*(c_H, S)$, recall that it maximizes $\pi_{2H}(p_2, c_H, S) = (p_2 - c_H)\min[T - S, (N_1 + N_2 - S)\overline{G}(p_2 - H)]$. Apply a change of variables: let $p_2' = p_2 - c_H$ and $H' = H - c_H$, and we have $\pi_{2H}(p_2, c_H, S) = p_2' \min[T - S, (N_1 + N_2 - S)\overline{G}(p_2' - H')] = \pi_{2H'}(p_2', 0, S)$, which is maximized at $p_2' = p_{2H'}^*(0, S)$. Thus, $p_{2H}^*(c_H, S) - c_H = p_{2H'}^*(0, S)$. This also immediately implies $\pi_{2H}^*(c_H, S) = \pi_{2H'}^*(0, S)$.

For $p_{1H}^*(c_H, S)$, by its definition, $p_{1H}^*(c_H, S) = \mathrm{E}[\min(p_{2H}^*(c_H, S), H + \alpha)] = \mathrm{E}[\min(p_{2H'}^*(0, S), H' + \alpha)] + c_H = p_{1H'}^*(0, S) + c_H$.

For $S_H^f(c_H)$, recall that it maximizes $\pi_H^f(c_H, S) = (p_{1H}^*(S) - c_H)S + \pi_{2H}^*(c_H, S)$. By the results proved above, $\pi_H^f(c_H, S) = \pi_{H'}^f(0, S)$. Hence, $S_H^f(c_H) = S_{H'}^f(0)$.

These results directly imply that Corollary 1, Lemma C.1, and Theorem 2 still hold for the modified model.

Lemma 1 holds, with the correction that p_{2H}^U maximizes the unconstrained profit $(p_2 - c_H)\overline{G}(p_2 - t)$ and

$$p_{2H}^U \begin{cases} \in (H + \underline{\alpha}, H + \overline{\alpha}) \text{ and is a solution to } p_{2H}^U = c_H + \frac{\overline{G}(p_{2H}^U - t)}{g(p_{2H}^U - t)} & \text{if } H - c_H < \bar{t} = \frac{1}{g(\underline{\alpha})} - \underline{\alpha} \\ = H + \underline{\alpha} & \text{if } H - c_H \ge \bar{t} \end{cases} \tag{S.16}$$

Theorem 1 holds, with the correction on the last two bullets: $H - c_H < \bar{t}$ and $H - c_H \ge \bar{t}$.

Lemma H.1 holds, with the correction that for H type, if $H - c_H \leq t^D(t)$, the seller sells in advance.

Asymmetric Information

Lemma D.1 holds, since by the results above, $\pi^*_{2H}(c_H, S) - \pi^*_{2L}(0, S) = \pi^*_{2H'}(0, S) - \pi^*_{2L}(0, S)$, which strictly decreases in S by Lemma D.1 for the original model.

Lemma 2: the proof follows exactly the same way as the proof of the lemma for the original model, with correction of the high type's cost. For example, to prove the first part of the lemma (i.e., the two iso-profit curves cross at most once), we only need to modify equation (13) to the following:

$$(p'_1 - c_H)S' + \pi^*_{2H}(S') = (p''_1 - c_H)S'' + \pi^*_{2H}(S'') = k_1 \tag{S.17}$$

with equation (14) remains the same:

$$p'_1 S' + \pi^*_{2L}(S') = p''_1 S'' + \pi^*_{2L}(S'') = k_2 \tag{14}$$

Subtracting equation (14) from equation (S.17), we get $\pi^*_{2H}(S') - \pi^*_{2L}(S') - c_H S' = \pi^*_{2H}(S'') - \pi^*_{2L}(S'') - c_H S''$. Because $\pi^*_{2H}(S) - \pi^*_{2L}(S)$ strictly decreases in S (Lemma D.1), so is $\pi^*_{2H}(S) - \pi^*_{2L}(S) - c_H S$. Hence, S' must equal to S''. The rest of the proof remains the same.

For the theorems, below we highlight the adaption of the original proofs for the modified problem.

Theorem 3:
(i) ($\Rightarrow$) Equations (10) and (11) jointly imply $\pi^*_{2H}(S^a_H) - \pi^*_{2L}(S^a_H) - c_H S^a_H \geq \pi^*_{2H}(S^f_L) - \pi^*_{2L}(S^f_L) - c_H S^f_L$, which further implies $S^a_H \leq S^f_L$ by Lemma D.1.
(iii-c) "if $S^a_H > 0$, $p^a_{1H} = p^*_{1H}(S^a_H)$": To prove that $(p^*_{1H}(\underline{S}), \underline{S})$ dominates (p^a_{1H}, S^a_H), note that

$$\begin{aligned}
&\pi^a_H(p^*_{1H}(\underline{S}), \underline{S}, 1) - \pi^a_H(p^a_{1H}, S^a_H, 1) \\
&= \pi^a_L(p^*_{1H}(\underline{S}), \underline{S}, 1) - \pi^*_{2L}(\underline{S}) + \pi^*_{2H}(\underline{S}) - c_H\underline{S} - [\pi^a_L(p^a_{1H}, S^a_H, 1) - \pi^*_{2L}(S^a_H) + \pi^*_{2H}(S^a_H) - c_H S^a_H] \\
&= \pi^a_L(p^f_{1L}, S^f_L, 0) - \pi^*_{2L}(\underline{S}) + \pi^*_{2H}(\underline{S}) - \left[\pi^a_L(p^f_{1L}, S^f_L, 0) - \pi^*_{2L}(S^a_H) + \pi^*_{2H}(S^a_H)\right] + c_H(S^a_H - \underline{S}) \\
&= \pi^*_{2H}(\underline{S}) - \pi^*_{2L}(\underline{S}) - [\pi^*_{2H}(S^a_H) - \pi^*_{2L}(S^a_H)] + c_H(S^a_H - \underline{S}) > 0.
\end{aligned}$$

(iii-d) For the case "$T \geq N_1 + N_2$ and $L < \bar{t}$", the two sub-cases are: $H - c_H < \bar{t}$ and $H - c_H \geq \bar{t}$.

Theorem 4: The definition of $D_H(S)$ changes to: $D_H(S) = \pi^a_H(p^*_{1H}(S), S, 1) - \pi^a_H(p^E_1, S^E, q) = p^*_{1H}(S)S + \pi^*_{2H}(S) - [p^E_1 S^E + \pi^*_{2H}(S^E)] - c_H(S - S^E)$.
Theorem 5: The last three conditions are: "$\bar{t} \leq L < H - c_H$", "$L < \bar{t} \leq H - c_H$", and "$L < H - c_H < \bar{t}$".
Theorem 6: The three cases are: "$L > t^D(T)$", "$L \leq t^D(t) < H - c_H$", and "$H - c_H \leq t^D(T)$".
Theorem 8: In the proof of the theorem, equation (S.14) becomes

$$\pi^*_{2H}(S^*) - \pi^*_{2L}(S^*) - c_H S^* \geq \pi^*_{2H}(S^f_L) - \pi^*_{2L}(S^f_L) - c_H S^f_L$$

Also, when proving $(p^*_{1H}(S'), S', 0)$ dominates (p^*_1, S^*, A^*), the equations are:

$$\begin{aligned}
&\pi^{a,AD}_H(p^*_{1H}(S'), S', 0, 1) - \pi^{a,AD}_H(p^*_1, S^*, A^*, 1) \\
&= \pi^{a,AD}_L(p^*_{1H}(S'), S', 0, 1) + \pi^*_{2H}(S') - \pi^*_{2L}(S') - c_H S' - \left[\pi^{a,AD}_L(p^*_1, S^*, A^*, 1) + \pi^*_{2H}(S^*) - \pi^*_{2L}(S^*) - c_H S^*\right] \\
&= \pi^*_{2H}(S') - \pi^*_{2L}(S') - c_H S' - [\pi^*_{2H}(S^*) - \pi^*_{2L}(S^*) - c_H S^*] > 0 \qquad \square
\end{aligned}$$

SC.3. Multiplicative Utility Function

We assumed so far a linear utility function $U = t + \alpha - p$. Another often used utility function is a multiplicative one: $U = \alpha t - p$ (e.g., Mussa and Rosen 1978, Moorthy and Png 1992, Desai et al. 2001), where α represents a customer's "intensity of taste for quality" or "marginal valuation per unit of quality". We show that all of our previous results hold for the multiplicative utility.

Theorem 10 *Suppose that a customer's utility function is $U = \alpha t - p$, $t = H$ or L. All the results in the original model, Theorem 1 through 8, hold for the multiplicative utility model.*

Proof of Theorem 10 We will first show that under the full-information case, $\frac{p^*_{2H}(S)}{p^*_{2L}(S)} = \frac{p^*_{1H}(S)}{p^*_{1L}(S)} = \frac{\pi^*_{2H}(S)}{\pi^*_{2L}(S)} = \frac{\pi^f_H(S)}{\pi^f_L(S)} = \frac{H}{L}$, and $S^f_H = S^f_L$. That is, when quality increases, for given S, the seller's prices and profits are proportionally scaled up, which implies that the optimal capacity ration remains unchanged. We will then use these results to prove all the lemmas and theorems.

Full Information

For $p^*_{2t}(S)$, recall that it maximizes $\pi_{2t}(p_2, S) = p_2 \min[T - S, (N_1 + N_2 - S)\overline{G}(p_2/t)]$. Apply a change of variables: let $p'_2 = p_2/t$, and we have $\pi_{2t}(p'_2, S) = tp'_2 \min[T - S, (N_1 + N_2 - S)\overline{G}(p'_2)] = \pi_{2\{t=1\}}(p'_2, S)$, which is maximized at $p'_2 = p^*_{2\{t=1\}}(S)$. Thus, $p^*_{2t}(S) = tp^*_{2\{t=1\}}(S)$. This immediately implies $\pi^*_{2t}(S) = t\pi^*_{2\{t=1\}}(S)$.

For $p^*_{1t}(S)$, by its definition, $p^*_{1t}(S) = \mathrm{E}[\min(p^*_{2t}(S), t\alpha)] = \mathrm{E}[\min(tp^*_{2\{t=1\}}(S), t\alpha)] = tp^*_{1\{t=1\}}(S)$. This implies that $\pi^f_t(S) = p^*_{1t}(S)S + \pi^*_{2t}(S) = t\pi^f_{\{t=1\}}(S)$.

For S^f_H, since it maximizes $\pi^f_t(S) = t\pi^f_{\{t=1\}}(S)$, $S^f_H = S^f_{\{t=1\}}$.

From the results above, we immediately have $\frac{p^*_{2H}(S)}{p^*_{2L}(S)} = \frac{p^*_{1H}(S)}{p^*_{1L}(S)} = \frac{\pi^*_{2H}(S)}{\pi^*_{2L}(S)} = \frac{\pi^f_H(S)}{\pi^f_L(S)} = \frac{H}{L}$, and $S^f_H = S^f_L$.

Note that these results directly imply that Lemma C.1, Theorem 1, Theorem 2, and Lemma H.1 still hold for the modified model.

Lemma 1 holds, with the correction that $\overline{G}(p^B_{2t}(S)/t) = \frac{T-S}{N_1+N_2-S}$ and p^U_{2t} maximizes the unconstrained profit $p_2\overline{G}(p_2/t)$ and

$$p^U_{2t} \begin{cases} \in (t\underline{\alpha}, t\overline{\alpha}) \text{ and is a solution to } p^U_{2t} = \frac{\overline{G}(p^U_{2t}/t)}{g(p^U_{2t}/t)} & \text{if } t < \bar{t} = \frac{1}{\underline{\alpha} g(\underline{\alpha})} \\ = t\underline{\alpha} & \text{if } t \geq \bar{t} \end{cases} \tag{S.18}$$

Corollary 1 holds, with the correction of $\overline{G}(p^*_{2t}(S) - t)$ to $\overline{G}(p^*_{2t}(S)/t)$.

Asymmetric Information

Lemma D.1 holds, since $\pi^*_{2H}(S) - \pi^*_{2L}(S) = (\frac{H}{L} - 1)\pi^*_{2L}(S)$, which strictly decreases in S since $\pi^*_{2L}(S)$ strictly decreases in S.

Lemma 2 and Theorems 3 through 8 all hold for the modified model and all the proofs for the original model remain valid for the modified model.