The Hong Kong University of Science and Technology
Dept of Information Systems, Business Statistics
and Operations Management
Dept of Industrial Engineering & Decision Analytics
Joint Seminar Announcement

Tight Guarantees for Multi-unit Prophet Inequalities and Online Stochastic Knapsack

by

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Abstract: Prophet inequalities are a useful tool for designing online allocation procedures and comparing their performance to the optimal offline allocation. In the basic setting of k-unit prophet inequalities, the magical procedure of Alaei (2011) with its celebrated performance guarantee of $1-1/sqrt(k+3)$ has found widespread adoption in mechanism design and general online allocation problems in online advertising, healthcare scheduling, and revenue management. Despite being commonly used for implementing a fractional allocation in an online fashion, the tightness of Alaei's procedure for a given k has remained unknown. In this paper we resolve this question, characterizing the tight bound by identifying the structure of the optimal online implementation, and consequently improving the best-known guarantee for k-unit prophet inequalities for all $k>1$.

We also consider the more general online stochastic knapsack problem where each individual allocation can consume an arbitrary fraction of the initial capacity. Here we introduce a new "best-fit" procedure for implementing a fractionally-feasible knapsack solution online, with a performance guarantee of $1/(3+ e^(-2)) ~ 0.319$, which we also show is tight with respect to the standard LP relaxation. This improves the previously best-known guarantee of 0.2 for online knapsack.

Our analysis differs from existing ones by eschewing the need to split items into "large" or "small" based on capacity consumption, using instead an invariant for the overall utilization on different sample paths.

Finally, we refine our technique for the unit-density special case of knapsack, and improve the guarantee from 0.321 to 0.3557 in the multi-resource appointment scheduling application of Stein et al. (2020).

(Joint work with Jiashuo Jiang and Jiawei Zhang)

Bio: Dr Will Ma is an Assistant Professor of Decision, Risk, and Operations at the Graduate School of Business in Columbia University. He received his Ph.D. in 2018 from the MIT Operations Research Center, advised by David Simchi-Levi, and also spent a year as a postdoc at Google. His research interests include the analysis of online algorithms, data-driven modeling, and optimization theory, applied to revenue and supply chain management. Previously, Will has been a co-founder of Lunarch Studios, the start-up that launched the strategy game Prismata. He is also a former professional poker player, and designed the poker class which is taught annually at MIT.

All interested are welcome!
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